Student Instruction Sheet: Unit 2 Lesson 3

Linear Systems

Suggested time: 75 minutes

What's important in this lesson:
In this lesson you will learn how to solve for the point of intersection between two lines.

Complete these steps:

1. Read through the lesson portion of the package independently.
2. Complete any of the examples in the lesson.
3. Check your lesson answer with the lesson key your teacher has.
4. Seek assistance from the teacher as needed. If you have any questions about the examples.
5. Complete the 'Assessment and Evaluation' and hand-in for evaluation. Be sure to ask the teacher for any assistance when you are experiencing any difficulty.

<u>Hand</u>-<u>in</u> the following to your teacher:

1. The 'Student Handout'.
2. Assessment and Evaluation Sheet

Questions for the teacher:

Student Handout: Unit 2 Lesson 3

Vocabulary

Each equation in the system is actually a linear relation and will define a straight line. When we solve a system of equations we are finding the coordinates of the intersection point of the two lines. This is the point that is "common" to both lines.
Graphical methods use the graph to find the intersection point.
Algebraic methods use only the equations to find the x and y values for the common point but we need to remember it still applies to the graph we could draw.

If we look at a table of values we can find the common point as it has the same x and y values in each table.

$y = x + 6$

x	y
0	6
1	7
2	8
3	9
4	10

$y = -2x + 12$

x	y
0	12
1	10
2	8
3	6
4	4

$y = 3x - 3$

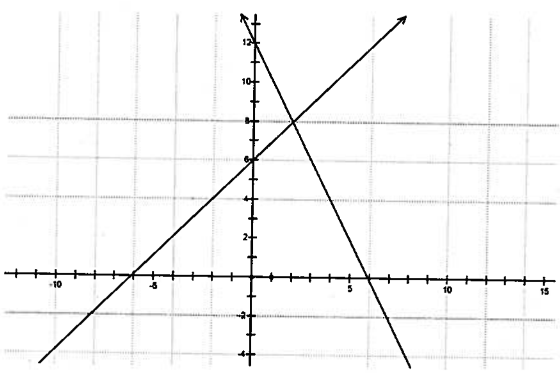
x	y
1	0
2	3
3	6
4	9
5	12

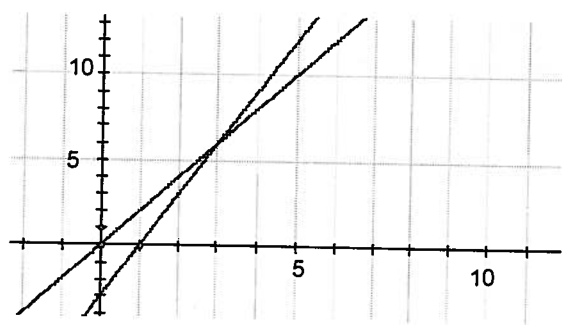
$y = 2x$

x	y
1	2
2	4
3	6
4	8
5	10

In this first example (2, 8) is on both lines. In this example (3, 6) is on both lines

When we graph two lines the solution is the point where the lines cross.





FILL IN → In this example (,) is the intersection of the system $y = x + 6$ and $y = -2x + 12$

In this example (,) is the intersection. of the system $y = 3x - 3$ and $y = 2x$

If we graph the lines accurately it doesn't matter if we see the common point in the tables because it will show up on the graph and can be taken from there if needed.

Solving By Graphing

To solve a system of equations by graphing, you must graph each line and look for the point of intersection.

Recall from previous work that to graph a line you must complete the following steps:

1. Find the slope (m) and y-intercept (b)
2. Plot the y-intercept on the graph
3. Use the slope to move according to the rise and run in order to plot a second point
4. Join the two points to form a line

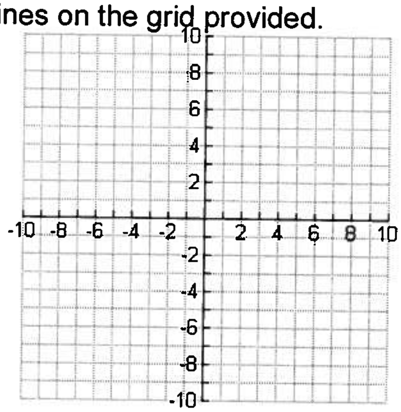
Practice:

1. Solve the system by graphing the lines on the grid provided.

$y = x - 4$

$y = -2x + 5$

Solution: (__,__)

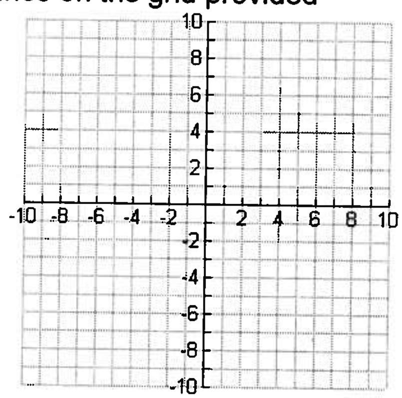


2. Solve the system by graphing the lines on the grid provided

$y = 3x - 6$

$y = \frac{1}{3}x + 2$

Solution: (__,__)

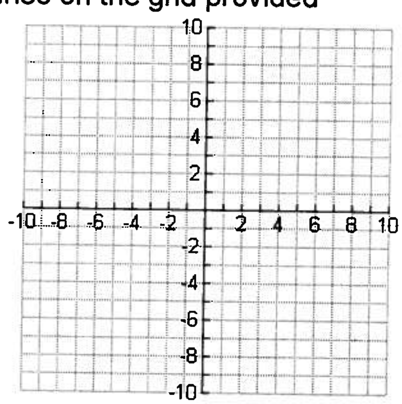


3. Solve the system by graphing the lines on the grid provided

$y = -3x - 6$

$y = \frac{1}{3}x - 6$

Solution: (__,__)



4. The solution to a system is more often called the ____________________

Student Handout: Unit 2 Lesson 3

<u>Algebraic Method 1 – Substitution</u> (This method is also called comparison if both equations are "y =" form)

Example 1.

Find the solution of :

$$y = 3x + 4 \quad (1)$$
$$2x + y = 9 \quad (2)$$

Step 1. Sub (1) into (2) (this means replace the "y" in equation 2 with the right hand side of the equation for y in 1.)

Step 2. Solve for x

$$2x + (3x + 4) = 9$$
$$5x + 4 = 9$$
$$5x = 9 - 4$$
$$5x = 5$$
$$x = 1$$

Step 3. Substitute the value of x from step 2 back into equation (1) to get y value.

$$y = 3\,(1) + 4$$
$$y = 3 + 4$$
$$y = 7$$

∴ the solution is (1,7)

Example 2.

Solve

$$y = 2x - 3 \quad (1)$$
$$x - 3y = -1 \quad (2)$$

Sub (1) into (2) to replace y

$$x - 3\,(2x - 3) = -1$$
$$x - 6x + 9 = -1$$
$$-5x = -1 - 9$$
$$-5x = -10$$
$$x = 2$$

Sub x = 2 into (1)

$$y = 2(2) - 3$$
$$y = 4 - 3$$
$$y = 1$$

∴ the solution is (2,1).

Algebraic Method 2 – Elimination (In this method we add or subtract the equations to eliminate (remove) one of the variables.

Example 1 Solve

$$2x + 5y = 17 \quad (1)$$
$$4x - 5y = -11 \quad (2)$$

Step 1 – Check to see if one of the variables can be eliminated by simply adding or subtracting the equations. In this case adding the equations will eliminate the variable y

Step 2 – Add or subtract the equations

$$\begin{array}{rl} & 2x + 5y = 17 \\ & \underline{4x - 5y = -11} \\ \text{Add} & 6x \qquad = 6 \end{array}$$

Step 3 - Solve for the remaining variable $x = 1$

Step 4 – Substitute the value found in step 3 in equation 1 to find the other variable

$$2(1) + 5y = 17$$
$$2 + 5y = 17$$
$$5y = 17 - 2$$
$$5y = 15$$
$$y = 3$$

∴ the solution is (1,3)

Example 2

Solve

$$x + y = -1 \quad (1)$$
$$3x - 4y = 25 \quad (2)$$

Step 1 – In this case neither adding nor subtracting will eliminate a variable since none of the coefficients (numbers in front of the variable) are the same. We can multiply every term of one equation by a constant to make the variables the same.

Multiply equation 1 y 3 (to make the x coefficients the same in both eq.) $3x + 3y = - 3$

Step 2

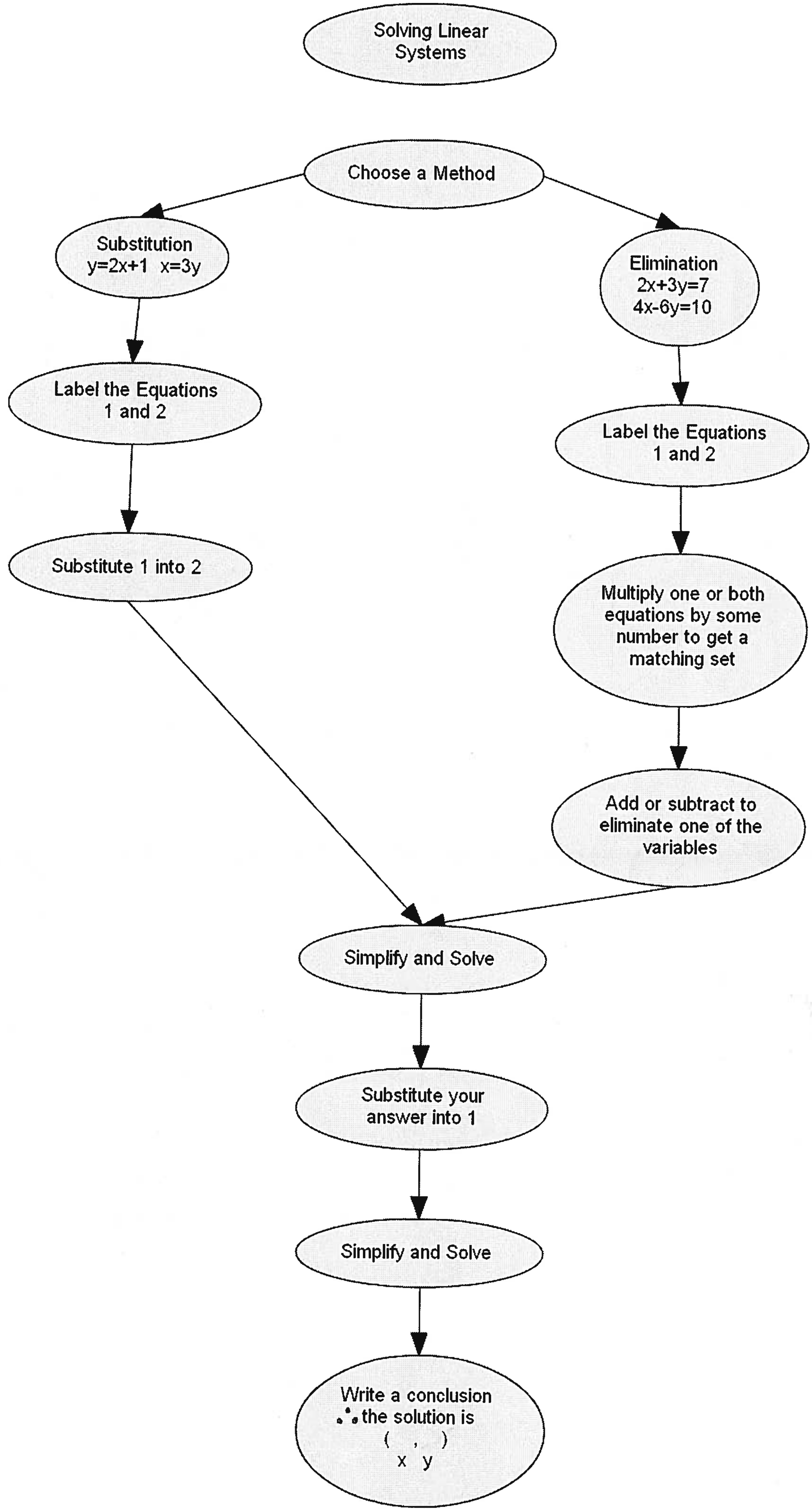
$$\begin{array}{rl} & 3x + 3y = - 3 \text{ (new } (1)) \\ & \underline{3x - 4y = 25} \\ \text{subtract} & 7y = - 28 \end{array}$$

Step 3 $y = - 4$

Step 4 Sub $y = - 4$ into equation 1

$$x + (- 4) = - 1$$
$$x - 4 = -1$$
$$x = - 1 + 4$$
$$x = 3$$

∴ the solution is (3,-4)


Solving Linear Systems
Choose a Method
Substitution
y=2x+1 x=3y
Elimination
2x+3y=7
4x-6y=10
Label the Equations 1 and 2
Label the Equations 1 and 2
Substitute 1 into 2
Multiply one or both equations by some number to get a matching set
Add or subtract to eliminate one of the variables
Simplify and Solve
Substitute your answer into 1
Simplify and Solve
Write a conclusion
∴ the solution is
(,)
x y

Practice:

1. Solve by substitution.

a) $y = 4x - 5$
$y = -2x + 7$

b) $y = 4x - 5$
$y = 3x + 7$

c) $y = 3x$
$y + 5x = 16$

d) $y = x - 3$
$2x + 3y = 6$

2. Solve by elimination.

a) $x + y = 4$
$2x - y = 8$

b) $2x + 3y = 5$
$2x - 3y = 3$

c) $x + 2y = 5$
$3x - y = 1$

d) $3x + 2y = 1$
$x - 3y = 4$

3. Solve using substitution or elimination.

a) $x + y = 7$
$5x + 3y = 15$

b) $y = 2x - 5$
$x + y = -2$

4. Solve and **check your answer in both equations**.

$7x - 3y = 17$
$7x + 3y = 11$

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1. State the solution to the given system of linear equations from the graph.

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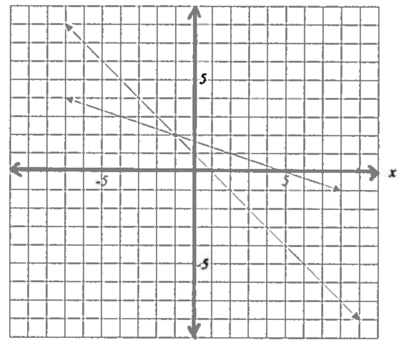
Solution: ____________________



2. Solve the following system by graphing both lines and finding the point of intersection.

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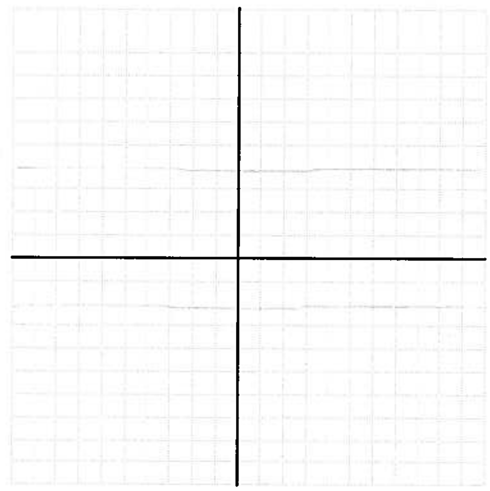
① $y = -2x + 6$

② $y = \dfrac{1}{2}x - 4$



6. Solve the following systems using either substitution or elimination.

a) ① $y = -2x - 4$

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② $y = 3x + 6$

b) ① $y = 2x + 1$
② $2x + y = -7$

$\overline{5}$

c) ① $2x - 6y = 4$
② $2x + 3y = 13$

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d) ① $x - 3y = 2$
② $2x + y = 11$

$\overline{5}$