Student Instruction Sheet: Unit 4 lesson 1

Similar Triangles

Suggested time: 75 minutes

What's important in this lesson:
In this lesson you will learn how to solve similar triangles.

Complete these steps:

1. Read through the lesson portion of the package independently.
2. Complete any of the examples in the lesson.
3. Check your lesson answer with the lesson key your teacher has.
4. Seek assistance from the teacher as needed. If you have any questions about the examples.
5. Complete the 'Assessment and Evaluation' and hand-in for evaluation. Be sure to ask the teacher for any assistance when you are experiencing any difficulty.

Hand-in the following to your teacher:

1. The 'Student Handout'.
2. Assessment and Evaluation Sheet

Questions for the teacher:

Student Handout: Unit 4 Lesson 1

Vocabulary

Two figures are **congruent** if they are exactly the same size and shape.

Two figures are **similar** if they are the same shape but different sizes.

The symbol "~" is a short form which means "is similar to".

The word **corresponding** is used a lot in this lesson. The meaning might be obvious but you need to make sure that you understand how it is being used. A simpler word for "corresponding" could be "matching". If we were comparing two triangles, each of them would have a shortest side, a middle side and a longest side. Taking the shortest side from each triangle would make a pair of corresponding sides. Similarly the largest angle from each triangle would make a pair of corresponding angles. If we were to take the ratio of the shortest side to the longest side within one of the triangles then we could also refer to the corresponding ratio in the other triangle.

A **proportion** is an equation which relates two equivalent ratios.

In this lesson we will investigate exactly how we can tell if two figures are similar. This will involve looking at three things.

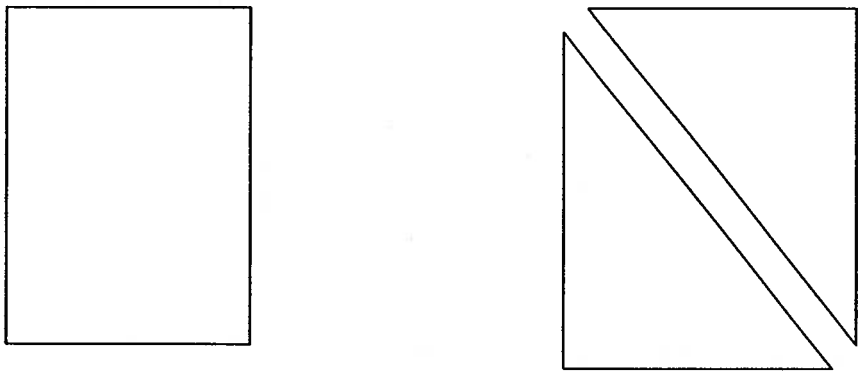
- the measure of the angles in the figure
- the ratio between the lengths of corresponding sides in the two figures
- the ratios of pairs of corresponding sides within the figures

You will need:

- one sheet letter size paper.
- protractor
- ruler
- scissors

1. Measure and label the side lengths on your piece of paper. Write a large signature across the back of your piece of paper. (You may need this later.)

2. Each rectangle has two diagonals. Fold your paper along one of the diagonals. Cut the paper along the diagonal.



3. What do you notice about the two triangles that you have created?

4. Take one of the two congruent triangles and set it aside. Take the other one and using a ruler and protractor draw a line that is perpendicular to the hypotenuse and passes through the vertex of the right angle. Cut the paper along this line. You should now have three triangles.

   Label the vertices of each triangle with appropriate letters (Largest triangle is ΔABC, Middle triangle is Δ DEF, Smallest triangle is ΔGHJ.)

   Explore the relationship between the triangles by reorienting them and overlapping the three triangles so that corresponding angles are in the same place.

5. Identify any triangles that you think are similar. Explain.

6. Using a ruler and protractor complete the table below to determine whether the triangles are similar.

Triangle	Hypotenuse	Shortest side	Middle side	Angles
ΔABC				
Δ DEF				
ΔGHJ				

7. Complete the following calculations.

$$\frac{Length \cdot of \cdot hypotenuse \cdot of \cdot \Delta DEF}{Length \cdot of \cdot hypotenuse \cdot of \cdot \Delta ABC} = \qquad \frac{Length \cdot of \cdot hypotenuse \cdot of \cdot \Delta DEF}{Length \cdot of \cdot hypotenuse \cdot of \cdot \Delta GHJ} =$$

$$\frac{Length \cdot of \cdot shortest \cdot side \cdot of \cdot \Delta DEF}{Length \cdot of \cdot shortest \cdot side \cdot of \cdot \Delta ABC} = \qquad \frac{Length \cdot of \cdot shortest \cdot side \cdot of \cdot \Delta DEF}{Length \cdot of \cdot shortest \cdot side \cdot of \cdot \Delta GHJ} =$$

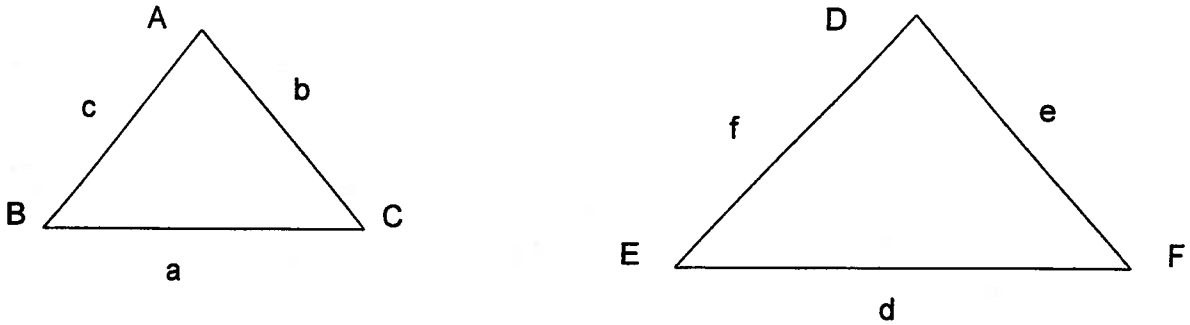
$$\frac{Length \cdot of \cdot middle \cdot side \cdot of \cdot \Delta DEF}{Length \cdot of \cdot middle \cdot side \cdot of \cdot \Delta ABC} = \qquad \frac{Length \cdot of \cdot middle \cdot side \cdot of \cdot \Delta DEF}{Length \cdot of \cdot middle \cdot side \cdot of \cdot \Delta GHJ} =$$

8. What do you notice about the ratios you have calculated in each column? State each ratio. **This ratio is called a scale factor.**

9. What conclusions about the triangles can you draw based on the ratios calculated in question 7? Are they similar or not? Explain.

Properties of similar triangles

1. If two triangles are similar then the corresponding angles must be equal.
2. The lengths of the corresponding sides form a proportion.



If the triangles above are **similar,** we could state the similarity by writing $\Delta ABC \sim \Delta DEF$ and the following statements must be true.

$$\angle A = \angle D$$
$$\angle B = \angle E$$
$$\angle C = \angle F$$

We can name sides using either the two capital letters for the endpoints or the small letter which matches the opposite vertex. This means we could write the proportion which relates the side lengths in two ways.

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} \quad \text{or} \quad \frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$

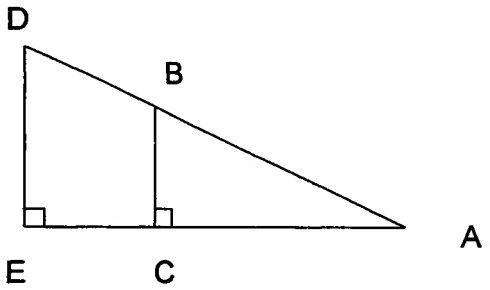
In these proportions we set up the side lengths from one triangle in the top line and then put the corresponding side from the other triangle underneath each one.

It's very important to realize that if **either one** of the two properties can be shown true for a pair of triangles, that's enough to guarantee that the triangles are similar and the other property must also be true. In other words if you measure the angles for two triangles and see that all the corresponding angles are equal then you don't have to check the side lengths because they are guaranteed to be proportional.
Or you could measure the sides to see that they are proportional and that would guarantee that the corresponding angles would have to be equal.

Student Handout: Unit 4 Lesson 1

In mathematics when we are asked to solve for the length of an unknown side in a diagram or situation we often need to identify two similar triangles and then set up and solve a proportion to find the length of the requested side. There are a few different ways that the similar triangles will often be set up.

Type 1



In this situation we have a right triangle nested inside a larger right triangle. From the properties of similar triangles we know that if two triangles have all corresponding angles equal then they must be similar and the corresponding sides must be proportional. In this case we know that Δ ABC ~ Δ ADE because

$\angle$ A is common to both triangles
$\angle$ C = $\angle$ E because both are labelled as right angles
$\angle$ B = $\angle$ D <u>either</u> because the two vertical lines are parallel <u>or</u> because all triangles have interior angles adding up to 180° so as soon as the first two pairs are shown equal the third pair will automatically be equal.

Once we are sure that the two triangles are similar we can write the proportion relating the side lengths.

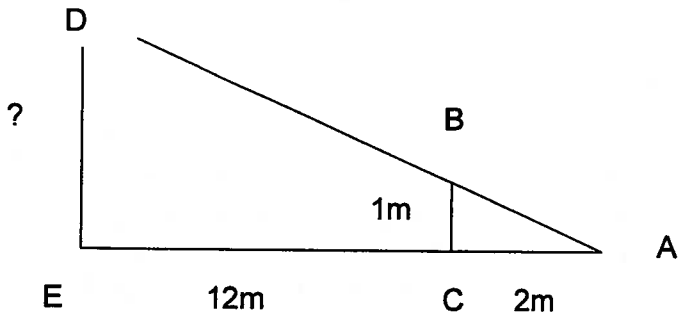
$$\frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE}$$

Even though this proportion can be used to relate all the pairs of corresponding sides we will often use only two of the fractions.
We will usually have **three** side lengths available and **one** side length requested so we will use the two fractions which match the given information.

Student Handout: Unit 4 Lesson 1

Example 1

A student wants to estimate the height of a tree which has a clearly visible shadow. She places a metre stick so that the shadow of the metre stick lines up exactly with the end of the shadow of the tree. She then uses a longer tape to measure the lengths of the shadows. She finds that the shadow of the metre stick is two metres long and the distance from the metre stick to the tree is 12 metres.



We know that the triangles ABC and ADE are similar so we can set up a proportion.

$$\frac{BC}{DE} = \frac{AC}{AE}$$

Now we substitute the given measurements to get

$$\frac{1}{DE} = \frac{2}{14}$$

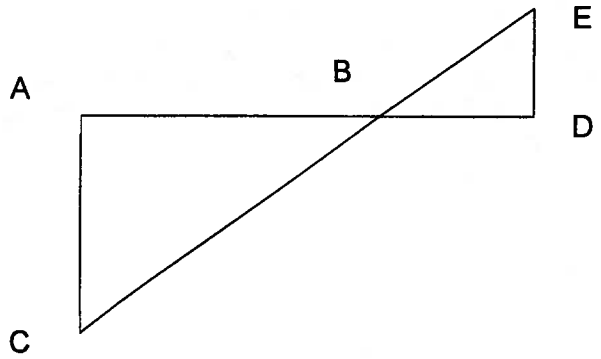
Notice that we had to add 12 + 2 to get the length for the base of the larger triangle!

Solving the equation

$$2\,DE = 14$$
$$DE = 14 \div 2$$
$$DE = 7$$

The height of the tree is 7 metres.

Type 2



In a diagram of this type we will be told or shown that the lines AC and DE are parallel. If these lines are parallel then we know that $\angle A = \angle D$ and $\angle C = \angle E$.
Because of the equal angle property we can conclude that $\Delta ABC \sim \Delta DBE$.

Because of the similarity we know that the sides must obey the proportion

$$\frac{AB}{DB} = \frac{AC}{DE} = \frac{BC}{BE}$$

Example 2

Assume that for a diagram such as the one above you had been told that
AB = 80m ; BD = 4m and DE = 3m. Solve for the length of AC.

Substituting into the proportion we get

$$\frac{80}{4} = \frac{AC}{3}$$

solving the equation

$$4\,AC = 240$$
$$AC = 240 \div 4$$
$$AC = 60$$

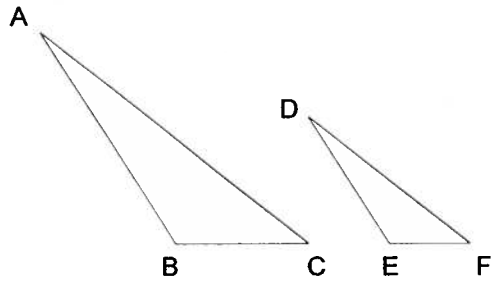
The requested length was 60 metres.

Practice:

1. For the following similar triangles, complete the two statements.

(a) $\frac{AB}{DE} = \frac{AC}{\_\_\_\_}$

(b) $\angle C = \_\_\_\_$

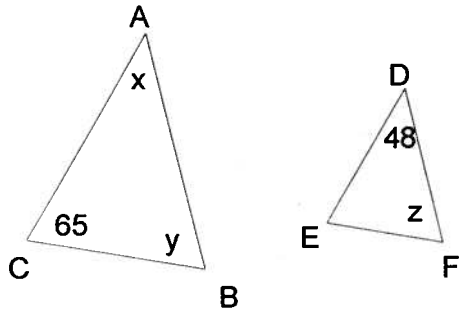


2. For the two similar triangles, find the missing angles.

(a) $x = \_\_\_\_$

(b) $y = \_\_\_\_$

(c) $z = \_\_\_\_$

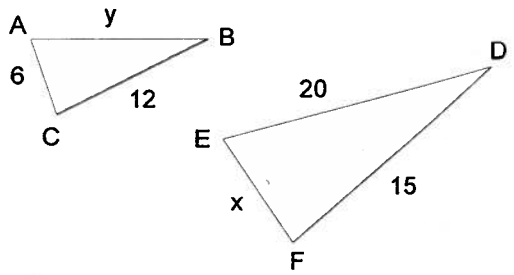


3. Solve the following proportions for x and y.

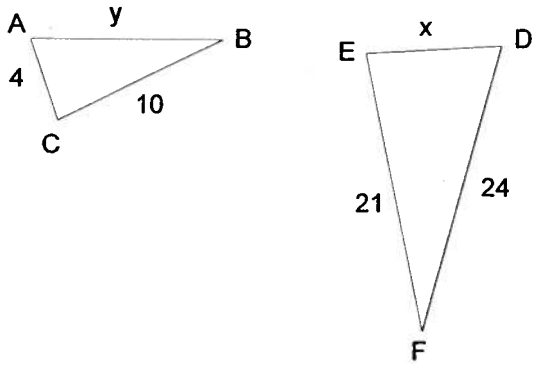
(a) $\frac{a}{6} = \frac{7}{21}$

(b) $\frac{2}{x} = \frac{13}{26}$

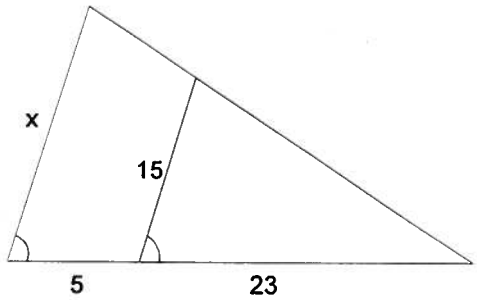
4. Find the unknown lengths using an appropriate proportion with similar triangles.



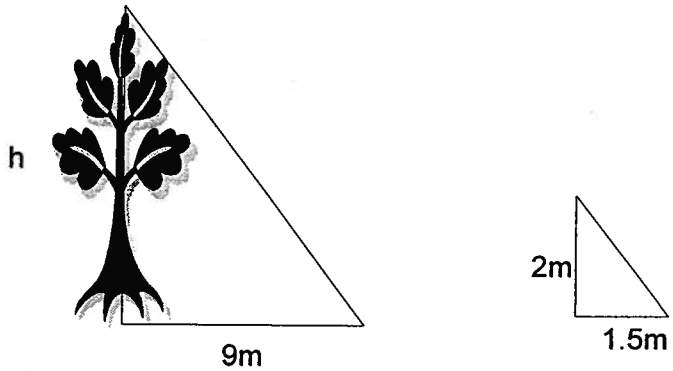
5. Find the unknown lengths using an appropriate proportion with similar triangles.



6. Find the missing side.



7. The shadow of a tree is 9m long at the same time of day that the shadow of a 2m man is 1.5m. Find the height of the tree.

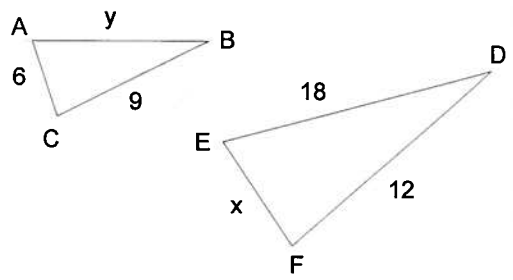


1. (4 marks) Solve the following proportions for x.

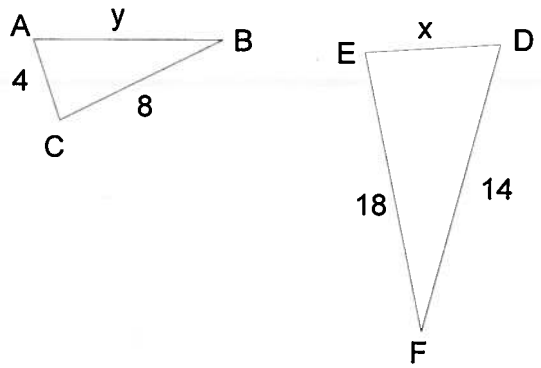
(a) $\frac{x}{28} = \frac{4}{16}$

(b) $\frac{3}{x} = \frac{12}{24}$

2. (6 marks) Find the unknown lengths using an appropriate proportion with similar triangles.



3. (6 marks) Find the unknown lengths using an appropriate proportion with similar triangles.



4. (4 marks) To determine the width of a river, surveyors placed stakes in the ground and made measurements as shown in the diagram. Determine the distance x across the river.