



Ratios of Sides of Right Angle Triangles

Suggested time: 75 minutes

What's important in this lesson:

In this lesson you will learn through investigation the relationship between the sides of a right triangle and the ratios of sine, cosine and tangent.

Complete these steps:

1. Read through the lesson portion of the package independently.
2. Complete any of the examples in the lesson.
3. Check your lesson answer with the lesson key your teacher has.
4. Seek assistance from the teacher as needed. If you have any questions about the examples.
5. Complete the 'Assessment and Evaluation' and hand-in for evaluation. Be sure to ask the teacher for any assistance when you are experiencing any difficulty.

Hand-in the following to your teacher:

1. The 'Student Handout'.
2. Assessment and Evaluation Sheet.

Questions for the teacher:



Background

In our previous lesson we worked with similar triangles. The focus in that lesson was on setting up ratios of sides between two similar triangles of different size. In this lesson we are setting up some definitions which allow us to use the idea of similar triangles applied to measured angles as well as just measured sides.

Calculator skills

The lesson below should help your understanding of exactly what the trigonometric ratios of an angle are. Before we explain them it will be useful in the investigation to know how to get a value for a trig ratio from your calculator.

Your calculator should have three buttons which are labelled sin, cos and tan. If it doesn't you'll have to find one that does!

Power up your calculator and hit the sin button. **If** you see the word "sin" appear in the display **then** follow these instructions to get the sine ratio of a given angle.

- press " sin " key
- input angle (your calculator should be set to degrees, get help from your teacher if necessary)
- press " = " key

You should see a decimal value which is bigger than 0 but smaller than 1.

To check see if you get a value of 0.5 for sin 30°.

If you got a "0" in the display after you hit the sin key then do this.

- input angle
- press "sin" key

You should see the decimal value right after you hit the "sin" key without having to hit the "=" key.



Definitions

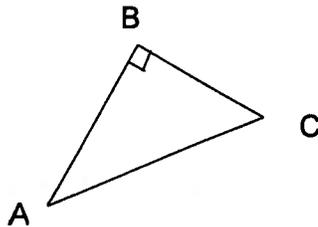
In a given right angle triangle there will always be two acute angles and a right angle. The acute angle which is labelled for the question we are looking at will be called the **reference angle**. You have to choose this angle!

In any right angle triangle the side which is across from the right angle is called the **hypotenuse**. This is always the longest side in the right triangle.

The side which joins the hypotenuse to form the reference angle is called the **adjacent side** of the right triangle.

The side which is across from the reference angle and forms the right angle with the adjacent side is called the **opposite side** of the right triangle.

Because we can draw right triangles in any orientation and choose either of two acute angles to be our reference angle the locations of the adjacent and opposite sides can't be predicted in terms of bottom, top, right or left. Always find the right angle first to label the hypotenuse. Then identify the reference angle to find the adjacent side and the opposite side will be the one left over.



If we choose $\angle A$ as our reference angle then
 AC is the hypotenuse
 AB is the adjacent side
 BC is the opposite side

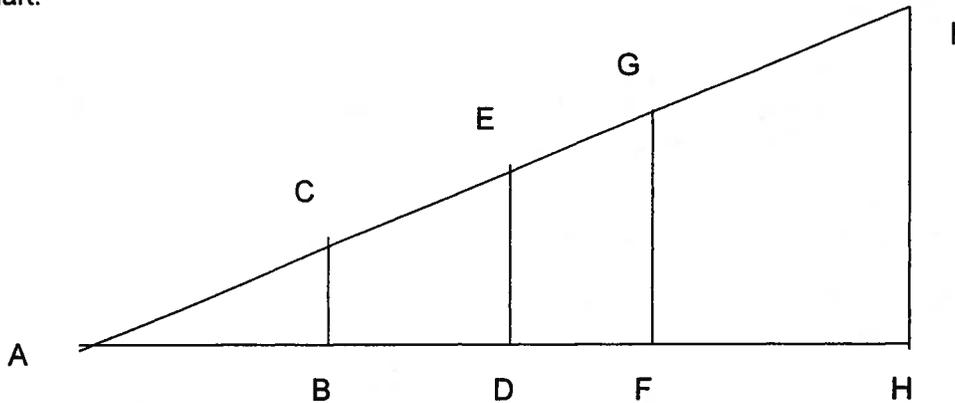
If we choose $\angle C$ as the reference angle then
 AC is still the hypotenuse but
 BC will be the adjacent side
 AB will be the opposite side

The sine, cosine and tangent ratios of an angle are called **trigonometric ratios**. These ratios identify the ratios of sides within any right triangle for a given measured reference angle.

Student Handout: Unit 4 Lesson 2



For the diagram below measure the requested lengths to the nearest mm and record in the chart.



Triangle	Length of Hypotenuse (mm)	Length of side adjacent to $\angle A$ (mm)	Length of side opposite to $\angle A$ (mm)	Measure of $\angle A$ ($^\circ$)
$\triangle ABC$				
$\triangle ADE$				
$\triangle AFG$				
$\triangle AHI$				

Now that we have recorded our lengths we can compare the ratios of different pairs of sides.

Triangle	$\frac{\text{opposite side length}}{\text{adjacent side length}}$
$\triangle ABC$	
$\triangle ADE$	
$\triangle AFG$	
$\triangle AHI$	

Using a scientific calculator find the value of the tangent ratio of the angle you measured for $\angle A$. How does this value compare to the values you have in your table?



Triangle	$\frac{\text{opposite side length}}{\text{length of hypotenuse}}$
ΔABC	
ΔADE	
ΔAFG	
ΔAHI	

Using a scientific calculator find the value of the sine ratio of the angle you measured for $\angle A$. How does this value compare to the values you have in your table?

Triangle	$\frac{\text{adjacent side length}}{\text{length of hypotenuse}}$
ΔABC	
ΔADE	
ΔAFG	
ΔAHI	

Using a scientific calculator find the value of the cosine ratio of the angle you measured for $\angle A$. How does this value compare to the values you have in your table?

The big idea that should come out of all this is that if you build a right triangle of any size at all using a specified measurement for the acute reference angle then the ratios between the sides will always be the same for that angle. These ratios are all stored in your calculator so you don't have to do the actual construction and measurement of the triangle.



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Definitions to memorize

For any right angle triangle with a specified acute reference angle –

The sine ratio of the angle is found by dividing the length of the side opposite to the reference angle by the length of the hypotenuse.

$$\text{sine ratio} = \frac{\text{opposite}}{\text{hypotenuse}}$$

The cosine ratio of the angle is found by dividing the length of the side adjacent to the reference angle by the length of the hypotenuse.

$$\text{cosine ratio} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

The tangent ratio of the angle is found by dividing the length of the side opposite to the reference angle by the length of the side adjacent to the reference angle.

$$\text{tangent ratio} = \frac{\text{opposite}}{\text{adjacent}}$$

For the sake of speed we usually just write these relationships as

$$\sin = \frac{\text{opp}}{\text{hyp}}$$

$$\cos = \frac{\text{adj}}{\text{hyp}}$$

$$\tan = \frac{\text{opp}}{\text{adj}}$$

"sew cǎ toc ăh"

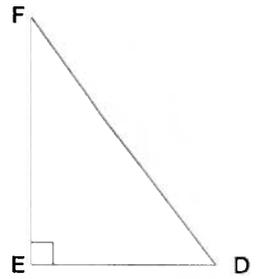
If we take the first letter from each word we can create a nonsense word **SOHCAHTOA** which some people use as a memory aid. When we do word problems involving trig we will always start by deciding which of the three ratios is needed for the problem we are being asked to do.

Steps:

- Circle the reference angle
- Label sides that have information
- Appropriate ratio chosen (sin, cos, tan)
- Substitute
- Solve

Practice:

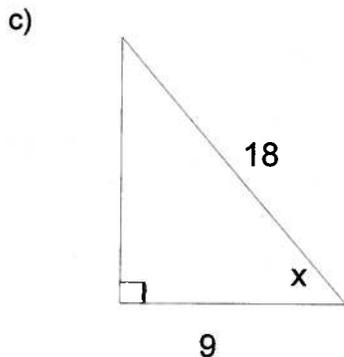
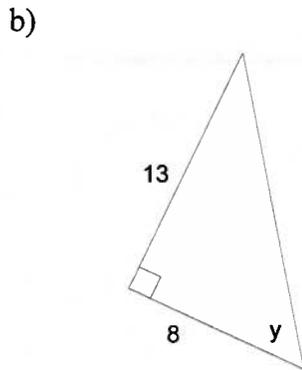
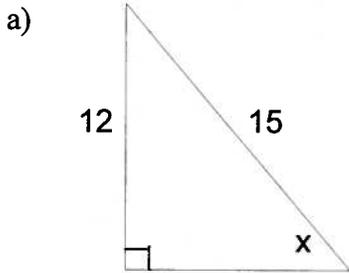
1. Given the angle F in the triangle DEF, label each side as either hypotenuse, adjacent, or opposite.

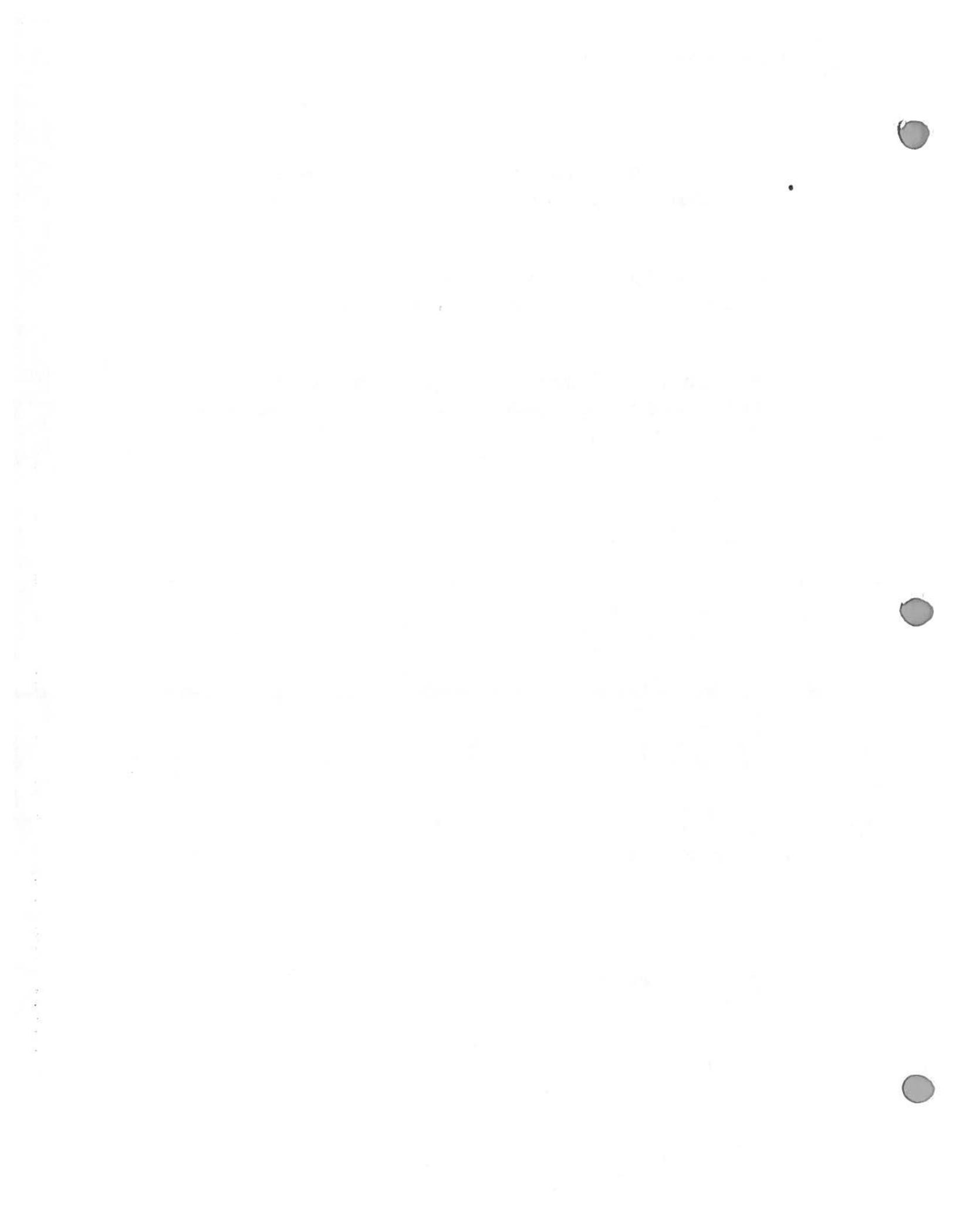


2. Evaluate each of the following to four decimal places.

a) $\sin 42^\circ =$ b) $\tan 18^\circ =$ c) $\cos 76^\circ =$

3. For each of the following circle the reference angle, label the sides that have information, choose a ratio (sin, cos or tan) and evaluate to four decimal places.







1. Use a scientific calculator to evaluate each of the following ratios. Round to 4 decimal places.

[a] $\sin 40^\circ =$

[b] $\cos 32^\circ =$

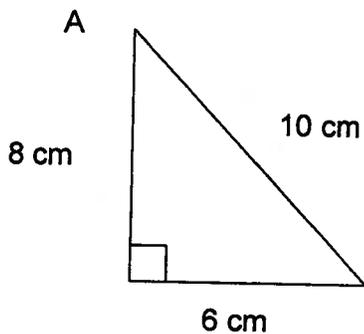
[c] $\tan 65^\circ =$

[d] $\cos 4^\circ =$

[e] $\tan 45^\circ =$

[f] $\sin 76^\circ =$

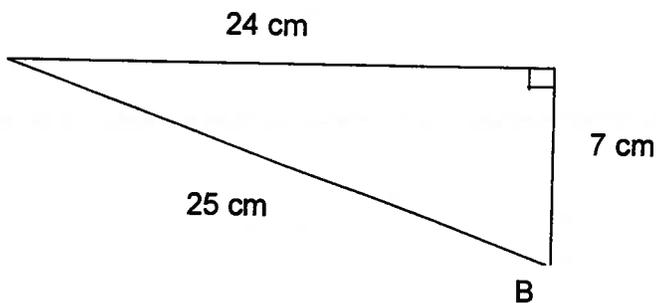
2. For the triangles below identify the adjacent side, opposite side and hypotenuse and use your definitions for the ratios to find the values requested. Round to 4 decimal places.



$\sin \angle A =$

$\cos \angle A =$

$\tan \angle A =$



$\sin \angle B =$

$\cos \angle B =$

$\tan \angle B =$

