



## Solving Problems using Trigonometric Ratios And Pythagorean Theorem

Suggested time: 75 minutes

### What's important in this lesson:

In this lesson you will solve problems using trigonometry (sin, cos, tan) and the Pythagorean theorem ( $a^2 + b^2 = c^2$ ).

### Complete these steps:

1. Read through the lesson portion of the package independently.
2. Complete any of the examples in the lesson.
3. Check your lesson answer with the lesson key your teacher has.
4. Seek assistance from the teacher as needed. If you have any questions about the examples.
5. Complete the 'Assessment and Evaluation' and hand-in for evaluation. Be sure to ask the teacher for any assistance when you are experiencing any difficulty.

### Hand-in the following to your teacher:

1. The 'Student Handout'.
2. Assessment and Evaluation Sheet

### Questions for the teacher:



## Vocabulary

An **angle of elevation** is an angle which is measured between the horizontal base of a right triangle and the hypotenuse. In a question of this type the hypotenuse will often be an imaginary sight line.

For example, if you are looking up at the top of a tower from the ground, then the horizontal base of the triangle is measured from you to the base of the tower and the angle of elevation is the angle between the ground and the imaginary line between you and the top of the tower.

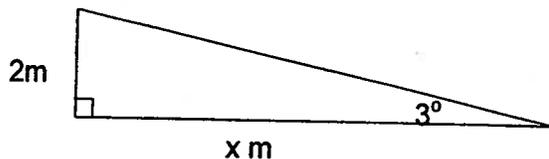
An **angle of depression** is an angle which is measured between an imaginary horizontal line drawn out from a high point and the hypotenuse of a right triangle joining the high point to some lower point.

## Important Note

To solve a problem with trigonometry we will always need a picture. Even if the question gives you a diagram, you should always draw a simple right angled triangle that shows the given and requested information. Then, follow the steps outlined in lesson 7 to solve for the unknown side or angle.

## Example 1

A ramp is to be built outside a building. The building code recommends the ramp must be at an angle of elevation of  $3^\circ$ . The entry of the building is 2m above the ground. How far from the base of the building will the ramp start?



The 2 m side is opposite to the  $3^\circ$  angle and the x m side is adjacent to the  $3^\circ$  angle. Because we have opposite and adjacent we have to use a tangent ratio for our equation.

$$\tan 3^\circ = \frac{2m}{xm}$$

$$x \tan 3^\circ = 2$$

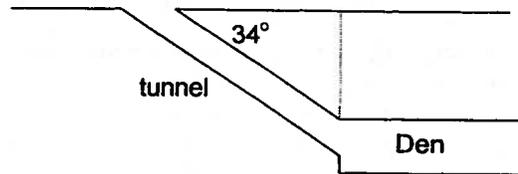
$$x = \frac{2}{\tan 3^\circ}$$

$$x \doteq 38.2m$$

Therefore the ramp must start about 38.2 m from the base of the building

**Example 2**

A mouse dug a tunnel at an angle of depression of  $34^\circ$ . The tunnel is 0.65m long with a small den at the end. How far below the surface is the den?

**Solution**

We choose the variable  $x$  to represent how far the den is below the surface.

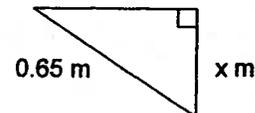
We redraw a simpler picture with labels:

$$\sin 34^\circ = \frac{x}{0.65}$$

$$0.65 \sin 34^\circ = x$$

$$x \approx 0.36$$

Therefore the den is about 0.36m below the surface.

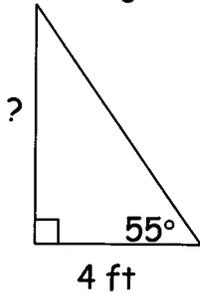


Our given side length is the hypotenuse, the requested side length is opposite the  $34^\circ$  angle. We must use the sine ratio.

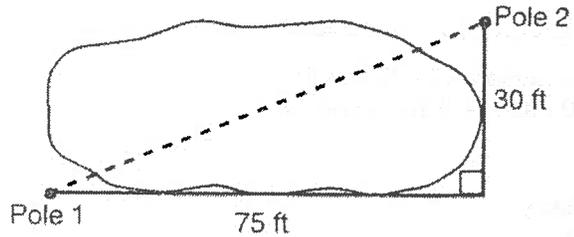
Practice:

Solve the application questions. Draw a diagram where necessary.  
 Find angles to the nearest degree and distances to the nearest tenth of a unit.

1. A ladder is leaning against a building and makes an angle of  $55^\circ$  with level ground. If the distance from the foot of the ladder to the building is 4 feet, find how far up the building the ladder will reach.



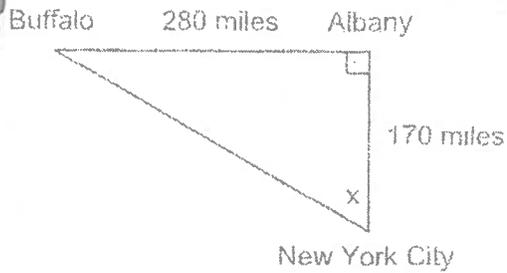
2. The Dodgers Communication Company must run a telephone line between two poles at opposite ends of a lake as shown below. The length and width of the lake is 75 feet and 30 feet respectively.



What is the distance between the two poles?

3. A ship on the ocean surface detects a sunken ship on the ocean floor at an angle of depression of  $50^\circ$ . The distance between the ship on the surface and the sunken ship on the ocean floor is 200 metres. If the ocean floor is level in this area, how far above the ocean floor is the ship on the surface?

4. Draw and label a diagram of the path of an airplane climbing at an angle of  $11^\circ$  with the ground. Find the ground distance the airplane has traveled when it has attained an altitude of 400 feet.



5. If an engineer wants to design a highway to connect New York City directly to Buffalo, at what angle,  $x$ , would she need to build the highway? Find the angle to the nearest degree.

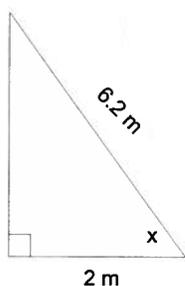
How many miles would be saved by travelling directly from New York City to Buffalo rather than by travelling first to Albany and then to Buffalo?

6. In order to safely land, the angle that a plane approaches the runway should be no more than  $10^\circ$ . A plane is approaching Pearson airport to land. It is at an altitude of 850 m. It is a horizontal distance of 5 km from the start of the runway. Is it safe for the plane to land?

7. An 8 m long ramp reaches up a vertical height of 1m. What angle does the ramp make with the ground?

8. A tree casts a shadow 42 m long when the sun's rays are at an angle of  $38^\circ$  to the ground. How tall is the tree?

1. (3 marks) A 6.2 m ladder leans against a wall. The foot of the ladder rests 2 m away from the wall. What is the angle formed between the ladder and the ground?



2. (4 marks) If you are standing 12 metres away from a smokestack and the angle of elevation to the top of the smokestack is  $53^\circ$ , how high is the smokestack? Draw a diagram as part of your solution.

3. (4 marks) A person is standing on top of a cliff that is 330 m high and sees a swimmer in the water below at an angle of depression of  $30^\circ$ . How far is the swimmer from the base of the cliff? Draw a diagram as part of your solution.

4. (5 marks) A person measures 1.5m from the ground to their eye level. Using a clinometer, they measure the angle of elevation to the top of a tree to be  $70^\circ$ . The person stands 17.5 m from the base of the tree. How tall is the tree? (Hint: you need to calculate  $x$  first, then  $h$ .)

